schedules would permit better utilization of class III, and would therefore produce better cost effectiveness.

It can be concluded that, with a relatively large space program, there is a role for post-Saturn. The exact size and composition of the program that economically justifies a large launch vehicle remain to be determined. The exact vehicle concept eventually selected will depend on the state of the art at the time of development initiation, but will probably include some degree of advanced propulsion and some form of recovery and re-use.

References

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Roll-Rate Lag of Rockets Accelerating in the Upper Atmosphere

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Nomenclature

A = aerodynamic reference area, exposed fin area, ft^2
B = aerodynamic reference length, fin semispan, measured from vehicle centerline, ft
C_l = rolling moment coefficient, rolling moment/qAB
C_{l_δ} = derivative of rolling moment with fin cant δ, rad^{-1}
C_{l_p} = damping derivative of rolling moment with fin tip helix angle pB/V, rad^{-1}
D = dimensionless fin cant driving parameter, rad^{-1}
= $C_{l_\delta}\delta\rho_{bo}AB^2/2\beta I \cos\gamma_{bo}$ (const)
Ei = exponential integral, $Ei\,(-x) = \int_\infty^x \left(\frac{e^{-x}}{x}\right)(dx)$, Eq. (10)
I = roll moment of inertia, slug-ft^2
$-K$ = $dV/dln\rho$, fps
M = Mach number
p = roll rate, rad/sec
q = dynamic pressure, $(\rho/2)V^2$, psf
Q = dimensionless roll damping parameter, rad^{-1}
= $-C_{l_p}\rho_{bo}AB^2/2\beta I \cos\gamma_{bo}$ (const)
t = time, sec
V = trajectory velocity, fps
β = exponential atmosphere scale factor; bove h = 35,000 ft
$1/\beta$ = 21,000 ft
γ = flight path angle, measured from the vertical, deg
Δ = increment
δ = fin cant, average of all fins, rad
ξ = fin tip helix angle, pB/V, rad
ρ = air density, slug/ft^3

Subscripts

bo = at burnout
ss = steady state
$(\ )'$ = derivative with respect to ρ/ρ_{bo}
$(\dot{\ })$ = derivative with respect to time

Introduction

THE results of a 6-D machine program, reported in Ref. 1, indicated that the estimated burnout roll rate for the Aerobee 350, based on the steady-state assumption, i.e.,

$$pB/V = C_{l_\delta}\delta/-C_{l_p}$$

was too high by a factor of 1.24. This does not include the effects of induced rolling moment, a phenomenon that is not treated herein. The roll-rate lag could cause significant effects on certain payloads that demand close control of the postburnout coning angle. Therefore, it becomes important to understand the causes of the burnout roll-rate lag so that compensating increments of fin cant may be prescribed to restore the roll rate to its desired level. This additional roll-rate requirement will accelerate the occurrence of pitch-roll resonance so that it will occur at a lower altitude and Mach number where the vehicle loads may be greater.

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Method of Analysis

The rolling moment differential equation of motion is

$$I\dot{p} = qAB[C_{l_\delta}\delta + C_{l_p}(pB/V)] \qquad (1)$$

This equation may be transformed into an easily solved differential equation by merely changing the independent variable from t to ρ. This is accomplished with the rate of climb relation

$$dh/dt = V \cos\gamma \qquad (2)$$

and the exponential atmosphere equation

$$\rho/\rho_{\text{sea level}} = e^{-\beta h} \qquad (3)$$

Equation (3) is differentiated and substituted into (2), yielding

$$d(p)/dt = -\beta V(\rho/\rho_{bo}) \cos\gamma d(p)/d\rho/\rho_{bo} \qquad (4)$$

Applying Eq. (4) to (1) gives

$$p'(B/V) = [-AB^2\rho_{bo}/2\beta I \cos\gamma][C_{l_\delta}\delta + C_{l_p}(pB/V)] \qquad (5)$$

Differentiating pB/V gives

$$(pB/V)' = p'(B/V) - (pB/V)(V/'V) \qquad (6)$$

Substitution of Eq. (6) in (5) produces the transformed differential equation of motion

$$\xi' + (V'/V - Q)\xi = -D \qquad (7)$$

which, assuming only that Q and D are constant, has the immediate integral solution

$$\xi = \left[\int_{\rho_{ss}/\rho_{bo}}^{\rho/\rho_{bo}} \left(\frac{V}{V}\right)_{bo} e - Q\rho/\rho_{bo}(-Q)d\left(\frac{\rho}{\rho_{bo}}\right)\right] \xi_{ss}\,\frac{eQ\rho/\rho_{bo}}{V/V_{bo}} \qquad (8)$$

where, by definition, $\xi_{ss} \equiv C_{l_\delta}\delta/(-C_{l_p}) = D/Q$.

Integration requires that the velocity be described as a function of the density. For sounding rockets accelerating swiftly in the upper atmosphere, a close description is given by $dV/dln\rho = -K$ (a constant). It can be shown that many types of rockets do indeed exhibit this linear, semilogarithmic variation, which exists for several thousands of feet below the burnout altitude, the region where nearly all of the dynamic roll lag occurs. At the lower altitudes, where this simple relation begins to fail, the steady-state analysis will apply so that the dynamic solution is not required. Although other analytic forms could be constructed, this one has an additional advantage in that it leads to a simple integration when it is employed in Eq. (8). Upon integrating it and satisfying the important burnout condition, the result is

$$V/V_{bo} = 1 - (K/V_{bo}) \ln(\rho/\rho_{bo}) \qquad (9)$$

Use of Eq. (9) in (8) permits integration in terms of the exponential integral. The integration is begun at the steady-state condition so that the initial steady-state conditions are satisfied:

$$\xi/\xi_{ss} = 1 + \frac{eQ\rho/\rho_{bo}}{(V_{bo}/K) - \ln(\rho/\rho_{bo})} \int_{Q\rho_{ss}/\rho_{bo}}^{Q\rho/\rho_{bo}} \frac{e^{-x}}{x}\,dx \qquad (10)$$

The values of this integral are tabulated, for example, in Ref. 3, as $Ei(-x)$. It is noted that $Ei(-x) < 0.01$ for values of the argument $Q\rho/\rho_{bo} > 3$, which will apply to nearly all values of the lower integral limit. Therefore, the integral in Eq. (10) may be written as $Ei(-Q\rho/\rho_{bo})$. For most purposes, the total history is not required, since only the burnout value is of interest. The special burnout value is simply

$$\xi_{bo}/\xi_{ss} = 1 + (K/V_{bo})e^{Q}Ei(-Q) \tag{11}$$

Equation (11) gives the burnout roll-rate lag, which is an important factor in specifying the required fin cant.

Results

The foregoing analytic expressions are somewhat unusual in that they are rigorously derived but are nevertheless of a simple form that is easy to use. The only assumption required in the integration was that the aerodynamic coefficients C_{l_δ} and C_{l_p} be constant. At high supersonic Mach numbers, the coefficient variation goes as $1/M$, which is indeed a slowly varying function. Since the largest part of the roll-rate lag occurs near burnout, it is suggested the burnout rather than a nominal value of C_{l_p} be used in the formula.

According to Eq. (11), the loss in roll rate is

$$\Delta\xi_{bo}/\xi_{ss} = (K/V_{bo})e^{Q}Ei(-Q) \tag{12}$$

Since $Q \ll 1$, Eq. (12) is approximated, for study purposes only, by

$$\xi_{bo}/\xi_{ss} = (-K/V_{bo})(1 + Q)\ln[1/1.781\,Q) \approx -(d\ln V/dh)_{bo}\ln(1/1.781\,Q) \tag{13}$$

This equation shows that the roll-rate lag varies directly as the logarithmic velocity change with altitude, whereas an increase in the damping coefficient Q will decrease the roll-rate lag.

Equation (12) was evaluated for the Aerobee 350, on which 6-D machine results[1] first uncovered the problem of roll-rate lag. For the case of a 150-lb payload with a two-caliber nose extension, these results[1, 2] apply:

M_{bo}	= 8.3	$1/\beta$	= 21,000 ft
V_{bo}	= 9080 fps	I	= 39 slug ft²
ρ_{bo}	= 3.71×10^{-6} slug/ft³	Q	= 0.174
h_{bo}	= 149,000 ft	$Ei(-Q)$	= −1.34
γ_{bo}	= 23 deg	$\Delta\xi/\xi_{ss}$	= −0.246, Eq. (15)
K	= 1400 fps		
C_{l_p}	= −0.317	$(\Delta\xi/\xi_{ss})_{6\text{-D}}$	= −0.233 (Ref. 1, Run 74)
AB^2	= 505 ft⁴		

This example clearly illustrates the close agreement between the analytic and 6-D machine runs.

When a specified burnout roll rate is required, the steady-state fin cant δ_{ss} should be given an increment of

$$\Delta\delta_{ss} = -\delta_{ss}(\Delta\xi_{bo}/\xi_{ss}) \tag{14}$$

to compensate for the roll-rate lag.

It could be argued that after burnout the roll rate will have time to accelerate back up to its steady-state value so that compensating fin cant will not be required. However, both the 6-D machine results and the analytic development[4] indicate that postburnout roll acceleration is negligible in the upper atmosphere.

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Application of Jet Pumping to Liquid Injection Thrust Vector Control

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SCHEMATICALLY, liquid injection thrust vector control (LITVC) system designs are simple. High-pressure gas (He, N_2, etc.) is regulated from a tank to the injectant tank; injectant is forced through, and is regulated by, the injector valves (Fig. 1). An electronic assembly provides command to the servo valves and the (hydraulically actuated) injectors. Nevertheless, a standard LITVC system may add appreciable weight (reactive-injectant, plus inert) to a stage; because high injection pressures are necessary, tanks, feed lines, and the injector constitute a large percentage of the total. In addition, valve actuation requires considerable power due to high line pressures; valve seating, chattering, and water hammer problems may be encountered. This note deals with decreasing weight by using jet-pumping for injection.

Jet-Pump Injection System

A specially designed jet-pump injector is shown in Fig. 2. Primary fluid expands through the nozzle at 1. Because of the low static pressure induced at 2, secondary fluid flows into the mixing chamber at 3. With the exchange of momentum and energy, the fluid mixture can be ejected with appreciable velocity and at a higher total pressure than either the primary or secondary injectant. From the conservation equations it can be shown that the mass-rate ratio of secondary to primary injectants is about 15, depending upon fluid property values.[1,2]

Figure 3 illustrates the principle of a monopropellant jet-pump system. Gas is regulated at high pressure to the small primary injectant tank and at low pressure to the large secondary tank. The valve allows 1) primary injectant to flow through the rocket section of the jet pump, and 2) secondary injectant to enter the mixing chamber. Primary advantages of the jet-pump system over the standard system are as follows:

1) The large secondary injectant tank (containing approximately 95% of the injectant weight) and the large diameter piping are maintained at a low pressure and are, therefore, light weight.

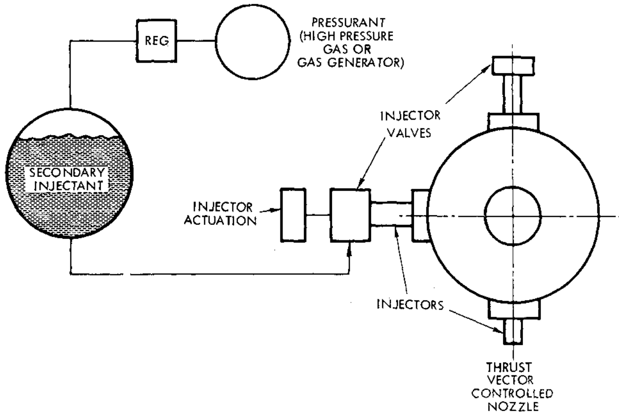

Fig. 1 Schematic: standard gas-pressurized LITVC system.

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